An Examination of the Performance of Variance Estimators in ILSAs

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Content

- Background
- Motivation for this study
- RQ's
- Methods
- Results
- Discussion

Why this research matters?

TIMSS PIRLS ICCS ICILS

Because **IEA studies** guide education policy,
our results must be **accurate and precise** to remain credible

Why uncertainty matters?

Weather forecast 🌤️
20 °C tomorrow

19–21 °C vs. **10–30 °C**
→ *very different forecasts*

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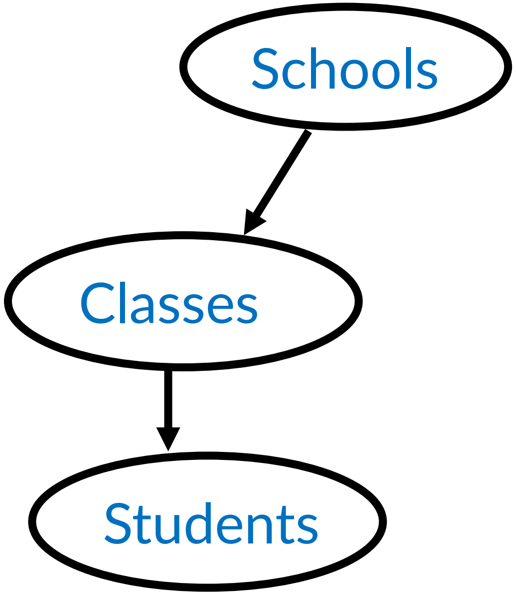
Uncertainty in estimates arises from **both assessment and sampling design** in IEA studies

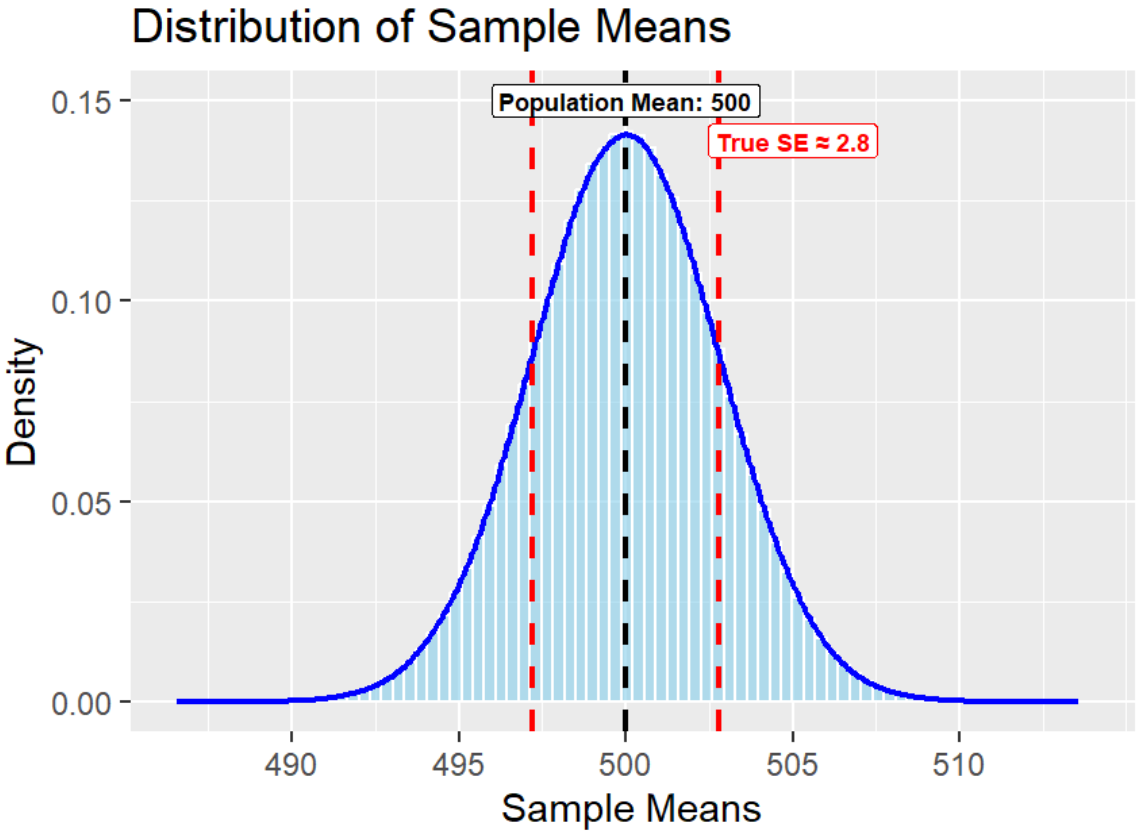
- Today, we focus on **sampling design**

Sampling Design in IEA Studies



- We carefully **sample schools and classes**
- We then assess **students** in the sampled classes
- Not all students or schools take part (**non-response**)
- **The final sample** is only one of many possible samples
- **Uncertainty** from these processes leads to **sampling error**

Focusing on the sampling error (SE)



- **SE** = the variation of results across all possible samples
- If we had all possible samples, we could compute **the true SE**
- With **only one sample**, we must estimate **the SE**
  - That is where **replication methods** come in

Replication Methods

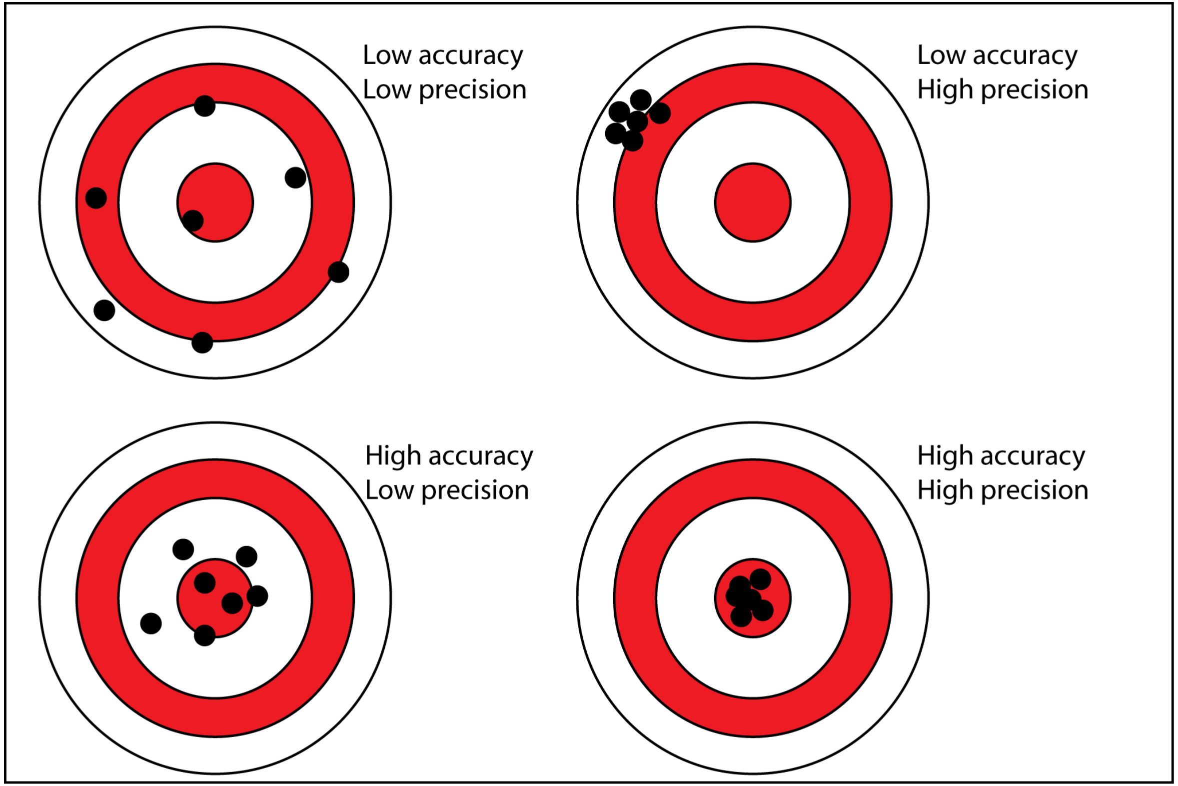
Replication ≈ resampling to approximate the sampling distribution

- Creates many sub-samples (replicates) from one sample
- Handles complex designs

Main approaches:

- **Jackknife Repeated Replication (JK2)**
  - ➢Used in IEA studies with
    - ▪ **Half** variant (e.g., 125 replicates)
    - ▪ **Full** variant (e.g., 250 replicates)
- **Balanced Repeated Replication (BRR)**
  - ➢Used in OECD studies, often with a Fay adjustment (e.g., 0.5)
- **Bootstrapping**
  - ➢Common in social sciences

Replication Methods – how do we compare them?



Accuracy → closeness to the true sampling error

Precision → consistency across samples

Gap in Literature

- No systematic comparisons of the replication methods in ILSAs
- Most evidence comes from other fields (e.g., Valiant et al., 2018)

Gap in Literature and Research Questions

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Which replication method works best for…

- Smooth statistics (means, correlations)
- Non-smooth statistics (percentiles, benchmarks)

How do methods perform under different conditions?

- Population backgrounds
- Sample sizes
- Non-response

Methodology

- 👥 **Population Design:** simulated a population with TIMSS-like characteristics
- 🔎 **Sampling Design:** two-stage stratified cluster sampling (schools and classes)
- 🖥️ **Simulation Procedure:** 1,000 samples per scenario (sampling distribution)
- 🧮 **Replication Methods (13):** JK2 (8), BRR (4), Bootstrapping (1)
- 📈 **Statistics**
  - **Smooth:** Means, subgroup differences, correlations
  - **Non-smooth:** Percentiles, benchmarks
- 🔢 **Total Iterations** about **1.56 million**

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Results – Highlights (1)

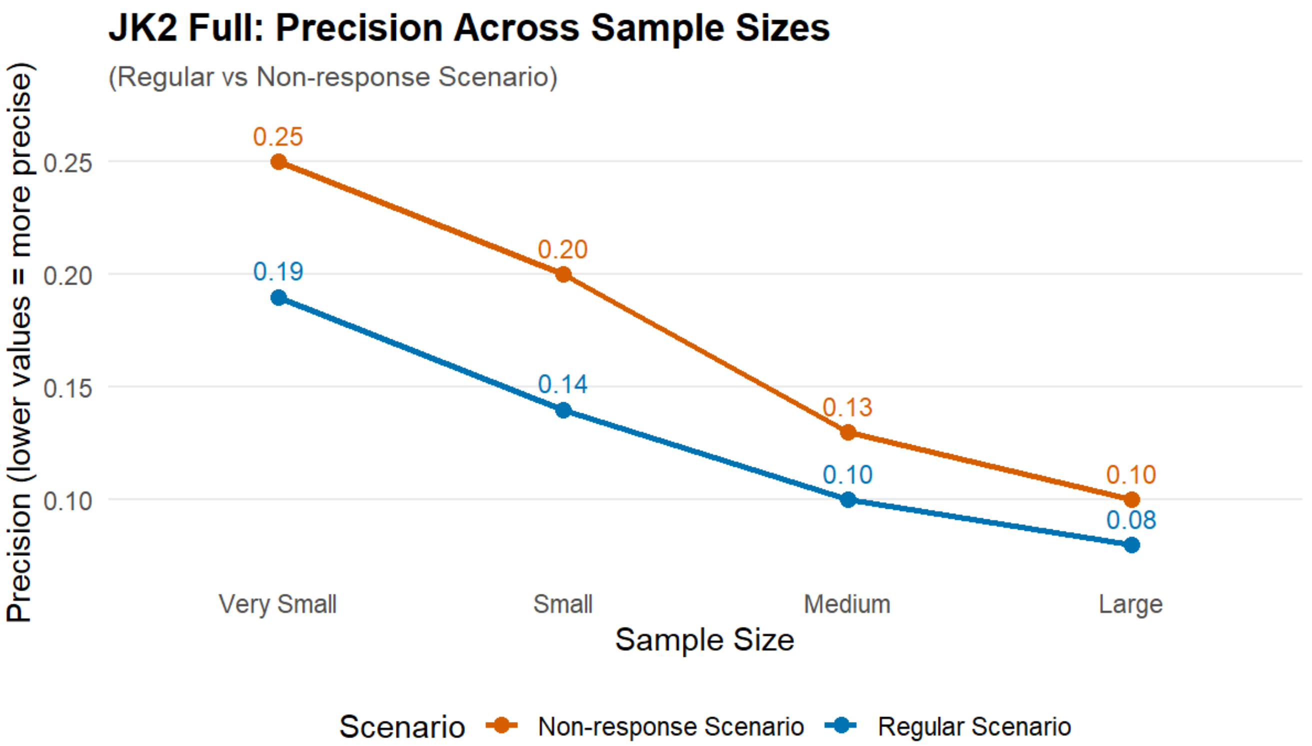
- **Methods perform well** under typical IEA sampling designs, including **JK2**, which we use in IEA studies
- **Smooth statistics** (means, correlations):
  - ➢**JK2 (Half or Full)** and **BRR** are both accurate and stable
- **Non-smooth statistics** (percentiles):
  - ➢**BRR** generally outperforms **JK2**
  - ➢**BRR with lower Fay factors (0.1)** performs much better than the **0.5 Fay** used in OECD studies
- **JK2 Half** performs same as **JK2 Full**
  → but requires less computation

Results – Highlights (2)

- **Non-response** reduces precision across all methods, especially in small samples
  → This is a real-world challenge and shows why **high participation is critical**

- **Re-pairing participating schools after non-response** improves accuracy
  → **IEA** uses this approach, while **PISA** keeps original order, which risks bias

Precision Under Non-response



Discussion – Key Takeaways

✅ **Practical guidelines** now exist to help IEA choose the most suitable replication methods

✅ **IEA replication methods are robust** — they deliver accurate and precise results under typical study conditions

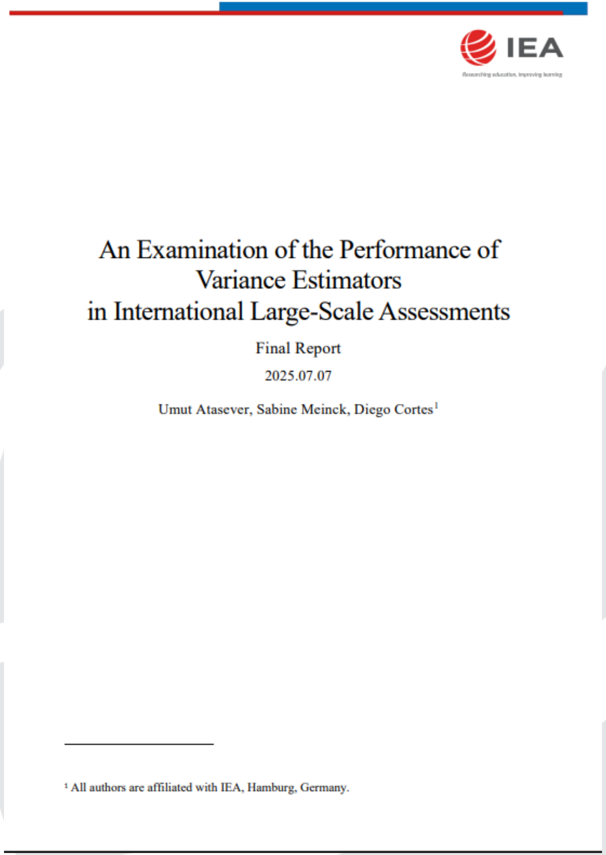
🚨 **Non-response is the biggest challenge** – reducing non-response is key to protecting accuracy and precision

👉 **Implication:**

- Method choice should depend on **statistic type** and **sample conditions**
- No one-size-fits-all, but **IEA approaches** have proven **reliable** across scenarios

The comprehensive report, with all scenarios and detailed results, can be found on the IEA website

QR Code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Researching education, improving learning

An Examination of the Performance of Variance Estimators in International Large-Scale Assessments

Final Report

2025.07.07

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